# Evaluating the Forecasting Performance of a DSGE-VAR Model for Romania

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***Abstract***
*Within the inflation targeting monetary policy regime, forecasts of central macro variables, inflation in particular, play an important part. Because inflation reacts to monetary measures with a considerable lag, the central bank's policy has to be forward-looking. This paper applies the DSGE-VAR methodology for a small open economy with an explicit inflation target – Romania, the aim being to assess the model's performance in terms of forecasting accuracy as opposed to a standard unrestricted VAR model. The results suggest that imposing information coming from a simple New Keynesian open economy model calibrated for Romanian economy as a prior in the estimation of a VAR helps improve the model's forecasting performance, especially in the medium term, for all the four variables considered in the VAR specification (real GDP, real exchange rate, inflation and nominal interest rate). Moreover, in contrast to the unrestricted VAR model, the DSGE-VAR model is informative about the structure of the economy and can help the "story-telling" in the central banks.*

*Keywords: Vector Autoregression, DSGE Model, Forecast*
*JEL codes: C32, C51, C53, F41*

## 1. Introduction

It is generally recognized that central banks policies must be forward looking, as there are long lags between monetary policy actions and their impact on the economy (Friedman, 1972). Therefore, macroeconomic forecasting has always been among the top priorities within central banks, the way in which forecasts are realized undergoing important changes over the past decades.

Since the seminal work of Sims (1980), vector autoregressions (VARs) have become an essential tool for forecasting in macroeconomics. The VAR is an econometric model used to capture the linear interdependencies among multiple time series, the only prior knowledge required being a list of variables which can be hypothesized to affect each other intertemporally. In theory, the idea is to let the data guide the views regarding the true data generating process. In practice, however, the parameters in the VAR models are often not very precisely estimated using classical econometrics procedures due to the dimensionality problem: high number of parameters to be estimated using a limited number of observations. Therefore, alternative methods for estimating the coefficients in a VAR model have been developed, the most successful being the Bayesian approach. The Bayesian estimation method provides a logical and formally consistent way of introducing shrinkage by treating the parameters of the model as random variables with probability distributions which are used to summarize the status of the knowledge about each parameter (prior information). By combining the prior information with the information contained in the data (the likelihood function) an updated distribution for the parameters is obtained, known as the posterior distribution, which is used to carry inference about the value of the parameters. Thus, to the extent that the prior is based on non-sample information, the Bayesian approach provides the ideal framework for containing different sources of information and thereby sharpening inference on macroeconomic analysis.

Even though the Bayesian VAR model is proven to be a reliable forecasting tool[1], the specific functions of a central bank imply the usage of models that are based on much more economic theory

[1] See, for example, Litterman (1980) and Kinal and Ratner (1986).

than a VAR model and are thus useful as a "story-telling" device. The large scale-models that were used by the central banks in the 1950s to 1970s were subject to strong criticism because of the lack of microeconomic foundations, which made them subject to the Lucas critique (Lucas, 1976), as well as ad hoc econometric restrictions. As a result a new class of models emerged, i.e. the dynamic stochastic general equilibrium (DSGE) models, built in recent years along the lines of New Keynesian Economics. The DSGE models are microfounded, having a consistent behavioural structure which helps interpretation. Moreover, the structural parameters that govern the relations between the variables in a DSGE model are invariant to changes in economic policy, so, in principle, not subject to the Lucas critique. However, the empirical evidence shows that DSGE models forecasts are usually dominated by univariate or multivariate time series[2] and, therefore, many central banks are still reticent in adopting a DSGE model as the main tool for supporting the policy making.

In their seminal works, DeJong et al. (1993) and Ingram and Whiteman (1994) respectively present an estimation methodology that unifies the two approaches mentioned above. DeJong et al. (1993) examine the impulse response functions generated by a VAR model estimated subject to the restrictions imposed by a monetary general equilibrium model, while Ingram and Whiteman (1994) demonstrate that prior information from a real business cycle model helps improve the forecasting performance in the case of movements in consumption, output, hours and investment for the US economy.

Del Negro and Schorfheide (2004) significantly extend the earlier work: first, by showing how posterior inference for the VAR parameters can be translated into posterior inference for DSGE model parameters, secondly by constructing a VAR identification scheme for the structural shocks based on a comparison of the contemporaneous VAR responses to shocks with the DSGE model responses and, finally, by illustrating how a VAR with DSGE model prior can be used to predict the effects of a permanent change in the policy rule.

Lees et al. (2007) complement the analysis of a DSGE-VAR forecasting performance for the economy of New Zeeland along policy dimension: they use the estimated DSGE-VAR structure to identify optimal policy rules that are consistent with the Reserve Bank's Policy Targets Agreement.

Other empirical applications of the DSGE-VAR methodology include Liu et al. (2007) for South Africa, Watanabe (2009) for Japan, Bache et al. (2010) for the Norway, Warne et al. (2013) for euro area etc.

This paper uses a simple New-Keynesian open economy model, calibrated for the Romanian economy, a country with an explicit inflation-targeting monetary policy regime since August 2005, as a prior in the estimation of a VAR model, the aim of the exercise being to show that the resulting model is competitive in terms of forecasting performance when comparing it with a standard benchmark represented by an unrestricted VAR model consisting of four variables: real GDP, real exchange rate, inflation and nominal interest rate.

Under inflation targeting, although the forecasting process centres around future price developments, the assessment of all the available information, including real economy and external balance developments, is also important. Also the open communication of monetary policy to the public is of crucial importance, as it influences the stability and predictability of monetary policy transmission into the economy. In this context, the proposed specification can also be used for policy analysis, as it retains part of the economic structure in the DSGE model, and is helpful as a "story-telling" device.

To the author's knowledge, this paper is the first to apply the DSGE-VAR methodology to the Romanian data.

The remainder of the paper is organised as follows: Section 2 contains a brief description of the DSGE model used to construct the prior distribution for the VAR model and outlines the DSGE-VAR methodology; Section 3 presents the data, while Section 4 compares the out-of-sample forecasts from the DSGE-VAR model with the ones obtained using the unrestricted VAR model. Concluding remarks are made in Section 5.

[2] See, for example, Del Negro and Schorfheide (2013).

## 2. The Methodology

This section briefly describes the DSGE model specification used to derive the prior distribution for the VAR model and outlines the DSGE-VAR methodology, following Del Negro and Schorfheide's paper (2004).

### *2.1 The DSGE Model*

The theoretical framework belongs to the New Neoclassical Synthesis paradigm, which builds on the Real Business Cycle literature, but includes also a series of elements characteristic of Keynesian models, the most important being the monopolistic competition, the nominal rigidities and the short run non-neutrality of monetary policy respectively. The model is adopted from Rabanal and Rubio-Ramirez (2005) and is extended to a small open economy setting.

There is a continuum of small open economies represented by the unit interval and indexed by k $\epsilon$ [0, 1]. Each small open economy is composed of: (i) a continuum of intermediate good producers, indexed by j $\epsilon$ [0,1], each producing a specific good that is an imperfect substitute for the other goods; (ii) a continuum of competitive final good producers; (iii) a continuum of infinitively lived households, indexed by i $\epsilon$ [0,1], each of them selling a type of labour that is an imperfect substitute for the other types; (iv) a monetary authority, without international policy coordination.

#### *2.1.1 Firms*

It is assumed that different economies share identical preferences, technology and market structure. Firms are identical across countries and have the same Cobb-Douglas production function[3]:

$$Y_{jt} = A_t N_{jt}^{1-\alpha} \tag{1}$$

where $Y_j$ is the output produced by firm $j$, $A_t$ is the economy-wide technology level and $N_j$ is an index of labor input used by firm $j$ and defined by a constant elasticity of substitution (CES) function that bundles the continuum of differentiated labour services provided by the households:

$$N_{jt} = \left[\int_0^1 \left(N_{jt}^i\right)^{\frac{\varepsilon_{w-1}}{\varepsilon_w}} di\right]^{\frac{\varepsilon_w}{\varepsilon_{w-1}}}. \tag{2}$$

α represents the capital share of output, while $\varepsilon_w$ denotes the elasticity of substitution among different labor types.

Price rigidity is introduced by using the staggered pricing rule of Calvo (1983). In any given period only a randomly chosen fraction of the firms $(1 - \Theta)$ are allowed to reoptimize their prices. The rest of the firms ($\Theta$) adjust their prices by partial indexation to previous period inflation. $\chi_p$ measures the degree of price indexation to last period's inflation.

Firm $j$ chooses its inputs and price in order to maximize the present value of its future profits.

#### *2.1.2 Households*

The lifetime utility function which a typical household $i$ seeks to maximize is additively separable in consumption, leisure and real money holdings respectively:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[ G_t^i \frac{\left(C_t^i\right)^{1-1/\sigma}}{1-1/\sigma} - \frac{\left(N_t^i\right)^{1+\gamma}}{1+\gamma} + \frac{\left(M_t^i/P_t\right)^{1-\nu}}{1-\nu} \right] \tag{3}$$

[3] Following McCallum and Nelson (1999), the capital stock is treated as fixed and investment is set to zero in the short run.

where $\beta$ is the discount factor, $\sigma$ is the elasticity of intertemporal substitution, $\gamma$ is the inverse of the elasticity of labour supply with respect to real wages and $\nu$ is the elasticity of money holdings with respect to transactions. $N^i$ denotes the labor services provided by the household *i*: each household specializes in one type of labour, which is supplied monopolistically. $M^i/P$ represents real money holding of household $i$, while $G^i$ is a preference-shifter shock that affects the marginal utility of consumption. The variable $C$ is a composite consumption index determined by both home and foreign goods[4]:

$$C_t = \left[(1-\varepsilon_B)^{\frac{1}{\varepsilon_H}}(C_{Ht})^{\frac{\varepsilon_H-1}{\varepsilon_H}} + \varepsilon_B^{\frac{1}{\varepsilon_H}}(C_{Ft})^{\frac{\varepsilon_H-1}{\varepsilon_H}}\right]^{\frac{\varepsilon_H}{\varepsilon_H-1}} \tag{4}$$

where $\varepsilon_B$ measures the degree of openness in the economy and $\varepsilon_H$ denotes the substitutability between domestic and foreign goods from the viewpoint of domestic consumers. $C_H$ is an index of consumption of domestic goods, given by the CES-function:

$$C_{Ht} = \left[\int_0^1 (C_{Hjt})^{\frac{\varepsilon_p-1}{\varepsilon_p}} dj\right]^{\frac{\varepsilon_p}{\varepsilon_p-1}} \tag{5}$$

where $\varepsilon_p$ represents the elasticity of substitution between varieties of goods produced in any country. $C_F$ is an index of consumption of imported goods:

$$C_{Ft} = \left[\int_0^1 (C_{kt})^{\frac{\varepsilon_F-1}{\varepsilon_F}} dk\right]^{\frac{\varepsilon_F}{\varepsilon_F-1}} \tag{6}$$

where $\varepsilon_F$ denotes the elasticity of substitution between importing countries. Finally, $C_k$ is an index of the different goods imported from country *k*:

$$C_{kt} = \left[\int_0^1 (C_{kjt})^{\frac{\varepsilon_p-1}{\varepsilon_p}} dj\right]^{\frac{\varepsilon_p}{\varepsilon_p-1}}. \tag{7}$$

The typical household *i*'s maximization problem is subject to a one-period budget constraint:

$$\begin{gathered}\int_0^1 P_{Hjt}C_{Hjt}^i\,dj + \int_0^1\int_0^1 (S_{kt}P_{kjt})C_{kjt}^i dj\,dk + M_t^i + \frac{1}{1+ir_t}B_t^i \\ \le M_{t-1}^i + B_{t-1}^i + W_t^i N_t^i + T_t^i\end{gathered} \tag{8}$$

The domestic price on good *j* is denoted $P_{Hj}$, the price on good *j* imported from country *k* and expressed in country's *k* currency is denoted $P_{kj}$, while $S_k$ represents the bilateral nominal exchange rate, i.e. the price of country *k*'s currency in terms of domestic currency. $B^i$ is the quantity of one-period nominally riskless bonds purchased each period by household $i$, which pay one unit of money at maturity and have a price of $\frac{1}{1+ir}$ units of money, $ir$ being the nominal interest rate. $W^i$ represents the nominal wage received for the type of labor provided by household *i*. Each period only a constant fraction of households $(1-\Theta_w)$ can reoptimize the price of their labour services, while for the remaining fraction of the households $(\Theta_w)$ the wage they had last period is adjusted by partial indexation to previous period inflation ($\chi_w$ measures the degree of wage indexation to last period's inflation). $T^i$ denotes lump-sum additions or subtractions to household $i$'s period income (taxes, dividends etc.).

[4] Aggregated together by the perfectly competitive final good producers.

*2.1.3 Monetary Authority*

It is assumed that the central bank follows a Taylor-type rule which allows for interest rate smoothing and penalizes the deviation of price inflation from the target $\pi$, as well as the output gap and the real exchange rate gap (*rer*).

*2.1.4 The Linearized Version of the Model*

To solve the model, optimality conditions are derived for the maximization problems. The dynamics of the model are obtained by taking a log-linear approximation around the steady-state equilibrium. The log-linearized system can be reduced to the following equations[5], where the small letter variables represent the log of large letter variables:

- Euler equation:

$$c_t = E_t[c_{t+1}] - \sigma(ir_t - E_t[\pi_{t+1}]) + \sigma(1-\rho_g)g_t \tag{9}$$

- Aggregate production function:

$$y_t = a_t + (1-\alpha)n_t \tag{10}$$

- The desired marginal rate of substitution ($mrs$) between consumption and hours worked:

$$mrs_t = \frac{1}{\sigma}c_t + \gamma n_t - g_t \tag{11}$$

- Taylor-type rule:

$$ir_t = \rho_r ir_{t-1} + (1-\rho_r)\left[\Phi_\pi \pi_t + \Phi_y y_t + \Phi_{rer} rer_t\right] + z_t \tag{12}$$

- Real wage growth rate:

$$w_t - p_t = w_{t-1} - p_{t-1} + (\Delta w_t - \pi_t) \tag{13}$$

- Phillips curve for domestic price inflation:

$$\pi_{H,t} = \gamma_b \pi_{H,t-1} + \gamma_f E_t\left[\pi_{H,t+1}\right] + \frac{\kappa_p}{1+\beta\chi_p}(w_t - p_t + n_t - y_t) \tag{14}$$

where $\kappa_p = \frac{(1-\alpha)(1-\Theta)(1-\beta\Theta)}{\Theta\left(1+\alpha(\varepsilon_p - 1)\right)}$, $\gamma_b = \frac{\chi_p}{1+\beta\chi_p}$ and $\gamma_f = \frac{\beta}{1+\beta\chi_p}$

- Phillips curve for domestic wage inflation:

$$\Delta w_t - \chi_w \pi_{t-1} = \beta E_t[\Delta w_{t+1}] - \beta\chi_w \pi_t + \kappa_w(mrs_t - w_t + p_t) \tag{15}$$

where $\kappa_w = \frac{(1-\Theta_w)(1-\beta\Theta_w)}{\Theta_w(1+\varepsilon_w\gamma)}$

- Demand for domestic goods:

$$y_t = (1-\varepsilon_B)c_t + \varepsilon_B y_t^* + \left(\varepsilon_B\varepsilon_H + \varepsilon_F\frac{\varepsilon_B}{1-\varepsilon_B}\right)rer_t \tag{16}$$

- Relationship between domestic inflation and CPI inflation:

$$\pi_t = \pi_{H,t} + \frac{\varepsilon_B}{1-\varepsilon_B}(rer_t - rer_{t-1}) \tag{17}$$

- Financial integration condition:

$$c_t = y_t^* + \sigma rer_t + \sigma g_t \tag{18}$$

$g_t$, $a_t$, $z_t$ and $y_t^*$ are a preference shifter shock, a technology shock, a monetary policy shock and a foreign demand shock. The evolution of these shocks is specified as follows:

$$g_t = \rho_g g_{t-1} + \varepsilon_t^g \tag{19}$$

$$a_t = \rho_a a_{t-1} + \varepsilon_t^a \tag{20}$$

$$z_t = \varepsilon_t^z \tag{21}$$

$$y_t^* = \rho_{y^*} y_{t-1}^* + \varepsilon_t^{y^*} \tag{22}$$

[5] Detailed derivations of the equations can be found, for instance, in Gali (2008).

where each innovation $\varepsilon_t^q$ follows a normal $\left(0,\sigma_q^2\right)$ distribution, $q=\{a,g,z,y^*\}$.

*2.1.5 Model Calibration*

The model is calibrated to match Romanian data (Table 1), following the results of the previous studies that apply the DSGE methodology for the Romanian economy (Viziniuc, 2013 and Copaciu et al., 2015).

Table 1: Calibrated parameters

| Parameter | Description | Value |
|---|---|---|
| β | Discount factor | 0.999 |
| α | Capital share in production | 0.45 |
| $\varepsilon_w$ | Elasticity of substitution among labour varieties | 11 |
| $\varepsilon_p$ | Elasticity of substitution between varieties of goods produced in any country | 4.5 |
| $\varepsilon_B$ | Degree of openness in the economy | 0.2 |
| $\varepsilon_H$ | Elasticity of substitution between domestic and foreign good in the domestic aggregate demand | 1.3 |
| $\varepsilon_F$ | Elasticity of substitution between importing countries | 1.3 |
| ϴ | Calvo parameter of price rigidity | 0.5 |
| $\Theta_w$ | Calvo parameter of wage rigidity | 0.6 |
| $\chi_p$ | Weight of price indexation to past inflation | 0.4 |
| $\chi_w$ | Weight of wage indexation to past inflation | 0.4 |
| $\Phi_\pi$ | Reaction coefficient to the deviation of price inflation from the target in the Taylor-type rule | 2.1 |
| $\Phi_y$ | Reaction coefficient to the output gap in the Taylor-type rule | 0.1 |
| $\Phi_{rer}$ | Reaction coefficient to the real exchange rate gap in the Taylor-type rule | 0.0 |
| $\rho_r$ | Inertia in the Taylor-type rule | 0.8 |
| σ | Elasticity of intertemporal substitution | 0.5 |
| γ | Inverse of the elasticity of labour supply with respect to real wages | 2.0 |
| $\rho_g$ | Persistence of demand shocks | 0.6 |
| $\rho_a$ | Persistence of productivity shocks | 0.6 |
| $\rho_{y^*}$ | Persistence of foreign demand shocks | 0.75 |
| $\sigma_g$ | Standard deviation of innovations in demand shocks | 0.3 |
| $\sigma_a$ | Standard deviation of innovations in productivity shocks | 0.7 |
| $\sigma_z$ | Standard deviation of innovations in monetary policy shocks | 0.3 |
| $\sigma_{y^*}$ | Standard deviation of innovations in foreign demand shocks | 2.5 |

Source: author's compilation

### 2.2 The DSGE-VAR Model

The DSGE-VAR approach implies the use of DSGE theory to construct prior beliefs over the VAR parameters.

The starting point for the estimation is an unrestricted VAR of order $l$:

$$y_t = A_0 + A_1 y_{t-1} + A_2 y_{t-2} + \cdots + A_l y_{t-l} + u_t, \tag{23}$$

where $t = 1, 2, \ldots, T$. $y_t = (y_{1t}, y_{2t}, \ldots, y_{nt})$ is a $n \times 1$ vector of observable variables, $A_0$ is a $n \times 1$ vector of constant terms, $A_1, A_2, \ldots, A_n$ are $n \times n$ matrices of autoregressive parameters and $u_t = (u_{1t}, u_{2t}, \ldots, u_{nt})$ is a vector of residuals following a multivariate normal distribution, i.e. $u_t \sim N(0, \Sigma_u)$.

T is the size of the sample used for estimation. A convenient reformulation of (23) consists in writing the VAR model in transpose form as:

$$y_t' = A_0' + y_{t-1}'A_1' + y_{t-2}'A_2' + \cdots + y_{t-l}'A_l' + u_t'. \tag{24}$$

Because (24) holds for any $t$, the model for the whole data set can be reformulated as:

$$Y = XA + U, \tag{25}$$

with $Y = \begin{bmatrix} y_1' \\ y_2' \\ \vdots \\ y_{T-1}' \\ y_T' \end{bmatrix}$, $X = \begin{bmatrix} 1 & y_0' & \cdots & y_{2-l}' & y_{1-l}' \\ 1 & y_1' & \cdots & y_{3-l}' & y_{2-l}' \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 1 & y_{T-2}' & \cdots & y_{T-l}' & y_{T-1-l}' \\ 1 & y_{T-1}' & \cdots & y_{T+1-l}' & y_{T-l}' \end{bmatrix}$, $A = \begin{bmatrix} A_0' \\ A_1' \\ \vdots \\ A_{l-1}' \\ A_l' \end{bmatrix}$ and $U = \begin{bmatrix} u_1' \\ u_2' \\ \vdots \\ u_{T-1}' \\ u_T' \end{bmatrix}$.

The likelihood function for the VAR model is given by:

$$p(Y/A, \Sigma_u) = |\Sigma_u|^{-T/2} exp\left\{-\frac{1}{2} tr[\Sigma_u^{-1}(Y'Y - A'X'Y - Y'XA - A'X'XA)]\right\} \tag{26}$$

Loosely speaking, implementing the prior from the DSGE model implies the augmentation of the dataset by a number of $T^* = \lambda T$ "artificial" observations $(Y^*, X^*)$ generated using the DSGE model, where λ is a hyperparameter representing the ratio of "artificial" data relative to the size of the actual sample of data. The prior distribution for the VAR parameters, according to the DSGE-VAR methodology, as presented by Del Negro and Schorfheide (2004), is based on the DSGE model representation as a reduced-form VAR[6], with the following likelihooh function:

$$p(Y^*(\xi)/A, \Sigma_u) = |\Sigma_u|^{-T/2} exp\left\{-\frac{1}{2} tr\left[\Sigma_u^{-1}\left(Y^{*\prime}Y^* - A'X^{*\prime}Y^* - Y^{*\prime}X^*A - A'X^{*\prime}X^*A\right)\right]\right\} \tag{27}$$

where $\xi$ represents the vector of structural parameters from the DSGE model.

The likelihood function for the combined sample of "artificial" and actual observations is obtained by premultiplying $p(Y/A, \Sigma_u)$ with $p(Y^*(\xi)/A, \Sigma_u)$, where the term $p(Y^*(\xi)/A, \Sigma_u)$ can be interpreted as a prior density for $A$ and $\Sigma_u$, of the Inverted Wishart (IW) – Normal (N) form, conditional on the vector of structural parameters $\xi$:

$$\begin{gathered} \Sigma_u/\xi \sim IW(\lambda T \Sigma_u^*(\xi), \lambda T - k, n) \\ A/\Sigma_u, \xi \sim N\left(A^*(\xi), \Sigma_u \otimes \left(\lambda T \Gamma_{XX}^*(\xi)\right)^{-1}\right) \end{gathered} \tag{28}$$

$A^*(\xi)$ and $\Sigma_u^*(\xi)$ are maximum likelihood estimators based on the sample of "artificial" data generated using the DSGE model:

$$\begin{gathered} A^*(\xi) = \left(\Gamma_{XX}^*(\xi)\right)^{-1} \Gamma_{XY}^*(\xi) \\ \Sigma_u^*(\xi) = \Gamma_{YY}^*(\xi) - \Gamma_{YX}^*(\xi)\left(\Gamma_{XX}^*(\xi)\right)^{-1} \Gamma_{XY}^*(\xi) \end{gathered} \tag{29}$$

The posterior distribution of the VAR parameters is also of Inverted Wishart-Normal form[7]:

$$\begin{gathered} \Sigma_u/Y, \xi \sim IW\left((\lambda+1)T\tilde{\Sigma}_u(\xi), (\lambda+1)T - k, n\right) \\ A/Y, \Sigma_u, \xi \sim N\left(\tilde{A}(\xi), \Sigma_u \otimes (\lambda T \Gamma_{XX}^*(\xi) + X'X)^{-1}\right) \end{gathered} \tag{30}$$

where

[6] Giacomini (2013) presents a literature review on the econometric relationship between DSGE and VAR models from the point of view of estimation and model validation.

[7] See Del Negro and Schorfheide (2004) for derivations and proofs.

$$\tilde{A}(\xi) = (\lambda T \Gamma^*_{XX}(\xi) + X'X)^{-1}(\lambda T \Gamma^*_{XY}(\xi) + X'Y) \tag{31}$$

$$\tilde{\Sigma}_u(\xi) = \frac{1}{(\lambda+1)T}[(\lambda T \Gamma^*_{YY}(\xi) + Y'Y) - (\lambda T \Gamma^*_{YX}(\xi) + Y'X)(\lambda T \Gamma^*_{XX}(\xi) + X'X)^{-1}(\lambda T \Gamma^*_{XY}(\xi) + X'Y)]$$

represent maximum likelihood estimates of $A$ and $\Sigma_u$ respectively, based on the combined sample of actual observations and "artificial" observations generated using the DSGE model.

$\lambda T \Gamma^*_{XX}(\xi)$, $\lambda T \Gamma^*_{XY}(\xi)$, $\lambda T \Gamma^*_{YX}(\xi)$ and $\lambda T \Gamma^*_{YY}(\xi)$, where for instance, $\Gamma^*_{XX}(\xi) = E_t[XX']$, denote the expected value at time $t$ of the sample of "artificial" observations moments $X^{*\prime}X^*$, $X^{*\prime}Y^*$, $Y^{*\prime}X^*$and $Y^{*\prime}Y^*$, i.e. the (scaled) population moments.

The hyperparameter λ governs the tightness of the prior distribution generated by the DSGE model for the parameters in the unrestricted VAR. In particular, setting $\lambda = 0$ delivers OLS-estimated VAR, i.e. DSGE prior is not important, while large $\lambda$ shrinks coefficients towards the DSGE solution, i.e. data is not important.

## 3. The Data

The empirical application uses Romanian quarterly data for real GDP, real exchange rate[8], inflation and nominal interest rate. The data is collected from the Romanian National Institute of Statistics database and National Bank of Romania database respectively. The sample covers the period from 2000 Q1 to 2014 Q4.

The variables are expressed as deviations from their trends, which are computed in the following way: for the output and the real exchange rate the Hodrick-Prescott filter is used, the smoothing parameter being set equal to 1600; in the case of inflation rate the trend is set equal to the inflation target[9]; for the nominal interest rate the trend is determined as the sum between the natural interest rate and the inflation target, where the natural interest rate is approximated by the average of the real interest rate taken on 2 sub-samples of data, before and after the economic crisis manifestation in the Romanian economy, i.e. the end of 2008.

## 4. Results

The DSGE-VAR model's forecasting performance is assessed against the unrestricted VAR model. The specification used includes four variables: real GDP, real exchange rate, inflation and nominal interest rate, which are expressed as deviations from their trends. The lag length in the VAR model is 2 quarters[10].

The forecasting experiment uses a rolling window estimation procedure which starts in 2000 Q1 and ends in 2012 Q4, the rolling window size being set to 8 years. Forecasts are computed for horizons of 1 to 8 quarters for each of the 20 rolling samples. The forecasting accuracy is measured by the root mean squared error (RMSE) indicator.

The results are presented in Table 2. In the case of the DSGE-VAR model the results are computed for different values of $p$, where $p \in [0, 1]$ denotes the weight given to the DSGE model, i.e. $p = \frac{\lambda T}{\lambda T + T}$, the notations being the ones introduced in section 2.

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[8] The exchange rate used in the estimation is an effective import-weighted exchange rate based on the bilateral exchange rates of the Romanian leu versus Eurozone and the United States of America respectively.

[9] Source: National Bank of Romania.

[10] In the existing literature (see, for example, Del Negro and Schorfheide (2004), Bache et al. (2007)), the lag length has usually been set to 4. However, due to the relative short sample of available observation, fixing the lag length to 2 quarters is appreciated as being a more appropriate approach.

Table 2: RMSEs for unrestricted VAR, DSGE-VAR and DSGE model

| Model | Weigh given to the DSGE model | Variable* | Horizon | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | T=1 | T=2 | T=3 | T=4 | T=5 | T=6 | T=7 | T=8 |
| Unrestr. VAR | $p = 0$ | Real GDP | 1.15 | 1.85 | 2.39 | 2.49 | 3.41 | 4.44 | 5.09 | 5.44 |
| | | Real exchange rate | 2.98 | 4.55 | 4.70 | 3.98 | 5.27 | 7.90 | 10.14 | 12.37 |
| | | Inflation rate | 2.93 | 3.36 | 3.41 | 3.49 | 3.79 | 4.24 | 4.53 | 5.22 |
| | | Nominal interest rate | 1.48 | 1.64 | 1.71 | 1.43 | 2.85 | 4.06 | 4.74 | 4.82 |
| DSGE-VAR | $p = 0.1$ | Real GDP | 1.12 | 1.72 | 2.18 | 2.34 | 3.32 | 4.37 | 5.11 | 5.65 |
| | | Real exchange rate | 2.94 | 4.33 | 4.49 | 4.05 | 5.18 | 7.57 | 9.57 | 11.62 |
| | | Inflation rate | 2.97 | 3.34 | 3.37 | 3.32 | 3.63 | 3.77 | 3.90 | 4.86 |
| | | Nominal interest rate | 1.29 | 1.34 | 1.43 | 1.32 | 2.37 | 3.11 | 3.75 | 3.68 |
| | $p = 0.4$ | Real GDP | 1.36 | 1.89 | 2.19 | 2.27 | 3.06 | 4.08 | 4.84 | 5.29 |
| | | Real exchange rate | 3.30 | 4.54 | 4.58 | 4.16 | 5.08 | 6.93 | 8.30 | 9.58 |
| | | Inflation rate | 3.32 | 3.48 | 3.47 | 3.47 | 3.49 | 3.65 | 3.89 | 4.70 |
| | | Nominal interest rate | 1.40 | 1.49 | 1.64 | 1.73 | 2.38 | 2.85 | 3.28 | 3.05 |
| | $p = 0.7$ | Real GDP | 1.67 | 2.15 | 2.34 | 2.32 | 2.86 | 3.76 | 4.47 | 4.82 |
| | | Real exchange rate | 3.92 | 4.95 | 4.71 | 4.31 | 5.29 | 6.88 | 7.81 | 8.51 |
| | | Inflation rate | 3.83 | 3.76 | 3.68 | 3.65 | 3.53 | 3.58 | 3.80 | 4.52 |
| | | Nominal interest rate | 1.77 | 1.84 | 2.02 | 2.21 | 2.75 | 3.08 | 3.35 | 3.03 |
| DSGE | $p = 1$ | Real GDP | 5.33 | 6.37 | 6.19 | 6.23 | 5.70 | 5.06 | 4.33 | 3.59 |
| | | Real exchange rate | 8.70 | 10.49 | 9.83 | 9.32 | 8.14 | 7.49 | 6.81 | 6.24 |
| | | Inflation rate | 5.92 | 5.38 | 4.90 | 4.16 | 4.03 | 3.63 | 3.38 | 3.61 |
| | | Nominal interest rate | 2.59 | 2.75 | 2.68 | 2.67 | 2.92 | 2.94 | 2.77 | 2.63 |

Note: *The variables are expressed as deviations from their trend.

Source: author's calculations

Based on these results, the DSGE priors prove to be useful as a mean of improving the forecasting performance of the VAR model. Short-term forecasting performance is higher for small values of $p$, while larger values of $p$ are preferred in the case of medium-term forecasting.

Although in the medium term the best results are obtained for the DSGE model, the DSGE-VAR model may still be a solution for the short-term forecasting in the situation that the DSGE model has been adopted as the main tool for supporting the policy making, as it allows to retain part of the economic structure in the DSGE model and provides better results in terms of forecasting performance than the DSGE model.

## 5. Conclusions

In the last decade, inflation targeting has been adopted by an increasing number of central banks as their monetary policy framework. Due to the delays in the monetary transmission mechanism, central banks with quantitative inflation targets, Romania included, must have adequate tools to form views on future macroeconomic performance, especially on inflation prospects.

The VAR models have long proven to be an effective method for modelling the dynamics of macroeconomic variables as well as forecasting. However, these models are atheoretical and use only the observed time series properties of the data to forecast economic variables, which makes them inappropriate for a central bank communication with the public and with the market.

This paper evaluates the forecasting performance of a DSGE-VAR model estimated on Romanian data, i.e. real GDP, real exchange rate, inflation and nominal interest rate, and compares it with the forecasting performance of an unrestricted VAR model. The results suggest that imposing prior information coming from a New Keynesian open economy model, calibrated for Romanian economy, in the estimation helps improve the forecasting performance of the VAR model, especially in the medium term. Moreover, in contrast to the unrestricted VAR model, the DSGE-VAR is informative about the structure of the economy and can help the “story-telling” in the central banks.

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**References**

BACHE, I.W., BRUBAKK, L., MAIH, J. (2010). *Simple rules versus optimal policy: what fits?* Norges Bank’s Working Paper 2010–03. Norway: Norges Bank.

CALVO, G. (1983). Staggered Prices in a Utility Maximizing Framework. *Journal of Monetary Economics*, vol. 12, no. 3, pp. 383–398.

COPACIU, M., NALBAN, V., BULETE, C. (2015). *R.E.M. 2.0. An estimated DSGE model for Romania*. Illinois: Northwestern University

DEJONG, D., INGRAM, B., WHITEMAN, C. (1993). Analysing VARs with Monetary Business Cycle Model Priors. In *Proceedings of the American Statistical Association, Bayesian Statistics Section*, American Statistical Association, pp. 160–169.

DEL NEGRO, M., SCHORFHEIDE, F. (2004). Priors from general equilibrium models for VARs. *International Economic Review*, vol. 45, no. 2, pp. 643–673.

DEL NEGRO, M., SCHORFHEIDE, F. (2013). DSGE Model–Based Forecasting. In *Handbook of Economic Forecasting* (edited by Elliott, G. and Timmermann, A.), vol. 2a, pp. 57–140.

FRIEDMAN, M. (1972). Have Monetary Policies Failed? *American Economic Review (Papers and Proceedings)*, vol. 62, pp. 11–18.

GALI, J. (2008). *Monetary Policy, Inflation and the Business Cycle. An introduction to the New Keynesian Framework*. Princeton: Princeton University Press.

GIACOMINI, R. (2013). *The relationship between DSGE and VAR models*. CEMMAP Working Paper 2013–CWP21. London: The Institute for Fiscal Studies.

INGRAM, B., WHITEMAN, C. (1994). Supplanting the ‘Minnesota’ prior. Forecasting macroeconomic time series using real business cycle model priors. *Journal of Monetary Economics*, vol. 34, pp. 497–510.

KINAL, T., RATNER, J. (1986). A VAR forecasting model of a regional economy: its construction and comparative accuracy. *International Regional Science Review*, vol. 10, no. 2, pp. 113–126.

LEES, K., MATHESON, T., SMITH, C. (2007). *Open economy DSGE–VAR forecasting and policy analysis: Head to head with the RBNZ published forecasts*. RBNZ Discussion Paper 2007–01. New Zealand: Reserve Bank.

LITTERMAN, R. (1980). *A Bayesian Procedure for Forecasting with Vector Autoregressions*. Minneapolis: Federal Reserve.

LIU, G., GUPTA, R., SCHALING, E. (2007). *Forecasting the South African Economy: A DSGE–VAR Approach*. University of Pretoria Working Paper 51. Hatfield: University of Pretoria.

LUCAS, R. (1976). Econometric Policy Evaluation: A Critique. In *Carnegie–Rochester Conference Series on Public Policy*, vol. 1, pp. 19–46.

MCCALLUM, B., NELSON, E. (1999). Nominal income targeting in an open–economy optimizing model. *Journal of Monetary Economics*, vol. 43, pp. 553–578.
RABANAL, P., RUBIO–RAMIREZ, J. (2005). Comparing New Keynesian models of the business cycle: A Bayesian approach. *Journal of Monetary Economics,* vol. 52, pp. 1151–1166.
SIMS, C. 1980. Macroeconomics and reality. *Econometrica*, vol. 48, no. 1, pp. 1–48.
VIZINIUC, M. (2013). Romania's accession to the Eurozone – a simulation using a simple DSGE model. *Theoretical and Applied Economics*, vol. 20, no. 8, pp.15–36.
WARNE, A., COENEN, G., CHRISTOFFEL, K. (2013). *Predictive likelihood comparisons with DSGE and DSGE–VAR models*. ECB Working Paper 1536. Frankfurt: European Central Bank.
WATANABE, T. (2009). *The Application of DSGE–VAR Model to Macroeconomic Data in Japan.* ESRI Discussion paper 225–E. Tokyo: Economic and Social Research Institute.