The Effect of Inflation Differential on the Nominal Exchange Rate: The Case of USA and Canada

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Abstract:
Macroeconomists from all over the world have always been interested in the influence of the inflation differential on the nominal exchange rate. This research paper focuses on the application of inflation differential theory to the US economy and one of its main trading partners - Canada. The primary aim of this paper was to prove that inflation differential affects the nominal exchange rate of the American dollar to the Canadian dollar. Secondary aim was to determine whether inflation differential and exchange rate depend on their previous values. This task was fulfilled by usage of VAR approach, which is considered to be standard econometric tool for stationary time series analysis. It was found out that inflation differential depends on its previous value, while exchange rate does not. Selected approach turned out to be unable to prove or disprove that inflation differential affects USD/CAD exchange rate. More variables should have been included in regression analysis, because inflation differential alone is not sufficient to explain changes in USD/CAD exchange rate.

Keywords: inflation differential, nominal exchange rate, time series, VAR
JEL codes: C30, C58, E31

1. Introduction

Inflation differential theory is closely connected with purchasing power parity theory (we refer to it as PPP in following reading) and with concept of real exchange rate. Inflation differential is defined as difference between inflation rates of two countries. According to CNB, real exchange rate is defined as the ratio of the price level abroad and the domestic price level, where the foreign price level is converted into domestic currency units via the current nominal exchange rate. The real exchange rate tells us how many times more goods and services can be purchased abroad (after conversion into a foreign currency) than in the domestic market for a given amount. Relationship among nominal exchange rate, real exchange rate, domestic price level and foreign price level is:

$$e = \varepsilon * \frac{P_d}{P_f} \tag{1}$$

where e denotes nominal exchange rate, ε denotes real exchange rate, P_d denotes domestic price level and P_f foreign price level.

Absolute form of PPP is linked to law of one price and works with assumption that the real exchange rate (ε) is equal to 1 and remains at this value under all circumstances. It means that value of domestic goods (consumer basket is usually used as benchmark) expressed in domestic currency is the same as the value of foreign goods (value of foreign goods is converted into domestic currency via nominal exchange rate). Therefore potential change in nominal exchange rate (e) must be caused by changes in price levels. We can rewrite equation (1) into following equation, which is known as above mentioned absolute PPP:

$$e = \frac{P_d}{P_f} \tag{2}$$

Relative form of PPP, which is usually used in empirical research, can be derived from its absolute form. We rewrite equation (2) as:

$$P_d = e * P_f, \tag{3}$$

which is related to the period *T*. In period T + 1 we have to take into account the changes of variables experienced during period T, so we get:

$$P_d * (1 + \pi_d) = e * (1 + \Delta e) * P_f * (1 + \pi_f), \tag{4}$$

where π_d denotes the domestic inflation rate, π_f denotes foreign inflation rate and Δe denotes change of nominal exchange rate (all of these variables are expressed in %). Now we take *e* from equation (2) and put it in equation (4):

$$P_d * (1 + \pi_d) = \frac{P_d}{P_f} * (1 + \Delta e) * P_f * (1 + \pi_f) \tag{5}$$

Variables P_d and P_f drop out from equation (5), and we get:

$$(1 + \pi_d) = (1 + \Delta e) * (1 + \pi_f) \tag{6}$$

Then we proceed as follows:

$$1 + \pi_d = 1 + \Delta e + \pi_f + \Delta e * \pi_f \tag{7}$$
$$\pi_d = \Delta e + \pi_f + \Delta e * \pi_f \tag{8}$$
$$\pi_d = \Delta e * (1 + \pi_f) + \pi_f \tag{9}$$
$$\pi_d - \pi_f = \Delta e * (1 + \pi_f) \tag{10}$$
$$\Delta e = \frac{\pi_d - \pi_f}{1 + \pi_f} \tag{11}$$

If we approximate equation (11), we get the relative form of PPP:

$$\Delta e = \pi_d - \pi_f \tag{12}$$

For additional mathematical background take a look into Barro (2010). Expression $\pi_d - \pi_f$ is above mentioned inflation differential and according to equation (12) it is equal to change of nominal exchange rate. In other words, from equation (12) is obvious that without interference of authorities to adjust inflation or (and) nominal exchange rate, currency of country with higher inflation rate will depreciate relatively to currency of country with lower inflation rate only by the value of inflation differential. If exchange rate adjusts to inflation differential, PPP states that real exchange rate stays at its value 1.

The logic behind this macroeconomic phenomenon is as follows (simultaneously we apply it directly to our selected countries – USA and Canada). If the inflation rate in USA is higher than in Canada, there will be a greater price increase in US goods than in Canadian goods. As the inflation rate differential increases in favor of US economy, US goods become more and more expensive (relatively to goods produced in Canada). Households are going to buy rather Canadian goods than goods produced in USA. To do so, they need to trade USD for CAD, which will lead to increase of USD supply and increase of CAD demand. Both of these factors will cause depreciation of USD relatively to CAD (and appreciation of CAD relatively to USD).

It needs to be noted that applicability of absolute form of PPP on real economies is very strong assumption and evidence of its existence has not been produced up to now. On the other hand, Burda and Wyplosz (2009) state that theory of relative PPP is backed by empirical evidence, however only in long term view. In short term, changes of nominal exchange rates are not solely driven by inflation differential. Empirical approach on general applicability of PPP on real economies can be found in

Canarella et al. (2014), Huang and Yang (2015) or Zhou and Kutan (2011). Validity of PPP in long run is researched in Chang et al. (2011) or Doğanlar et al. (2009). PPP in short run is investigated in Arize et al. (2015).

Macroeconomists have come up with several theories why it is not possible to explain nominal exchange rates changes only by inflation differential (in other words, why prices of goods can differ internationally). The main reason is that a lot of inputs entering the production process cannot be traded internationally (or at least cannot be traded easily because of high transaction costs – see Michael et al. (1997)) in order to achieve arbitrage and equalize price levels in domestic and foreign country – for example labor force or property. These inputs or goods are called non-tradable and they are the main reason why PPP does not hold. Effect of non-tradable goods on real exchange rate is content of study Betts and Kehoe (2008). Second reason is that goods are not identical, although they can have similar characteristics – they differ for example in quality. Typical example is case of cars.

Differences in statistical approaches of individual countries can also negatively affect practical application of PPP. For example consumer baskets vary over countries, as well as the methods of measuring the inflation rate.

As mentioned above, the aim of this paper is to find out whether relative PPP holds for USA and Canada or not, thus prove that change in USD/CAD exchange rate can be explained by inflation differential alone. If we fail to prove this hypothesis, attention on causes of failure will be paid in further research.

2. Methodology and Data

Our dataset consists of two pairs of data with different length and with different frequency. It was collected 188 observations of US and Canada CPIs and USD/CAD nominal exchange rate since January 2000 to August 2015, both on monthly basis. Secondly, it was collected 92 observations of US and Canada GDP deflators and USD/CAD exchange rate since third quarter 1991 to second quarter 2014, this time both on quarterly basis. All data are acquired via Bloomberg database and are processed in Gretl software. CPIs (GDP deflators) are defined as percentage change of price levels therefore value of inflation differential was obtained simply by taking differences between CPIs (GDP deflators). In order to obtain percentage changes of exchange rate, we took first differences of logged variable USD/CAD multiplied by 100. Thus our dataset consists of four variables, which are seasonally adjusted to avoid misleading regression results and are expressed as percentage changes. Table 1 contains a short description of variables and their abbreviations used in the analysis.

Table 1: Variables used in analysis

abbreviation of variable	**variable characteristic**
dif_CPI	inflation differential constructed on the basis of CPIs of USA and Canada by taking their differences (expressed in %)
rate_m	monthly percentage change of nominal exchange rate constructed as first differences of logged exchange rate USD/CAD multiplied by 100 (expressed in %)
dif_deflator	inflation differential constructed on the basis of GDP deflators of USA and Canada by taking their differences (expressed in %)
rate_q	quarterly percentage change of nominal exchange rate constructed as first differences of logged exchange rate USD/CAD multiplied by 100 (expressed in %)

Source: author's own elaboration

Because we use lagged variables in further analysis, these lagged variables are marked with suffix in form of number representing the length of lag, for example expression dif_CPI_1 means inflation differential constructed on the basis of CPIs lagged by one period, etc.

In order to save space in case of comments on achieved results, it was decided to shorten following expression:

- inflation differential constructed on the basis of CPIs of USA and Canada by taking their differences (expressed in %) to CPI differential, and
- inflation differential constructed on the basis of GDP deflators of USA and Canada by taking their differences (expressed in %) to deflator differential.

The analysis takes form of linear regression. All the above mentioned variables are naturally in the form of time series. Time series have some special properties, which can lead to invalid results of regression therefore special handling is required in order to achieve correctness of estimation. Before the regression is performed, isolated analysis of each time series is executed to check assumption of stationarity (or non-stationarity).

Procedure of testing possible non-stationary behavior can be found in every advanced econometric textbook. Procedure used in this paper is based on Koop (2008) and was already used in author's earlier paper (Urbanovský, 2015). Basic characteristic of nonstationary time series is presence of unit root. In order to test for existence of unit roots, we proceed from following equation:

$$Y_t = \alpha + \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \cdots + \phi_p Y_{t-p} + \varepsilon_t \tag{13}$$

In the above mentioned equation (13), presence of unit root would be demonstrated by a coefficients ϕ equal to 1. In discussing (or testing) unit root behavior it is convenient to subtract Y_{t-1} from both sides of the equation (13). We obtain:

$$\Delta Y_t = \alpha + \rho Y_{t-1} + \gamma_1 \Delta Y_{t-1} + \cdots + \gamma_{p-1} \Delta Y_{t-p+1} + \varepsilon_t, \tag{14}$$

where $\rho = \phi - 1$ and $\rho = 0$ imply that time series Y_t contains unit root and is non-stationary. On the other hand, if $\rho = 0$, term Y_{t-1} drops out from equation (14), it induces stationarity of series ΔY_t.

Statistical significance (or insignificance) of every single lag of dependent variable would be discovered simply by testing whether individual coefficients are equal to zero on chosen significance level or not – we take a look at p-values or t-statistics. In case of coefficient ρ we need to take a look at Dickey-Fuller test statistic and corrected p-value.

Because inflation differential is the difference between the domestic inflation rate and the foreign inflation rate and inflation rate itself is expressed in terms of percentage change of price level, we can suspect that this time series is stationary (differences usually tends to be stationary). Stationarity of differenced time series could be suspected from their graphs, where values are oscillating around zero (see Appendix 1). On the other hand, exchange rate is level variable, therefore could be non-stationary. In order to deal with this possible unwanted property, it was decided to induce stationarity by taking differences of exchange rate. In particular, we take natural logs of this time series, then take differences of these logged series, then multiply them by 100. That implies that we are working with percentage changes in exchange rate, as is noted in Koop (2009). If we have both variables in form of percentage changes, it simplifies the interpretation of regression results. We are also working with stationary time series, so VAR approach could be used. VAR is successfully used on issues of PPP, real exchange rate and inflation differential in study Malliaropulos et al. (2013). However, this issue can also be investigated from the view of nonstationarity and cointegration, as is done in Cerrato and Sarantis (2008) or Dimitriou and Simos (2013).

Following characteristics and steps of VAR procedure are adopted from another author's earlier paper (Urbanovský, 2015). VAR is a typical econometric tool for stationary time series analysis. It is a system of regression equations, where the number of equations matches the number of variables under study. In each equation we have different dependent variable – it is always one of the variables under study. Each equation uses as its explanatory variables lags of all variables. Because it would be time and space consuming to find specific number of lags for each variable, it was decided to use the same lag length for every variable in every equation (note that this is a common practice in research papers). The resulting model is known as a VAR(p) model with *p* indicating number of

included lags. Extensive theoretical background on VAR approach can be found for example in Greene (2012) or Heij (2004).
The research process has the following structure:

- finding appropriate number of lags,
- estimation of VAR(p) based on information received in previous step
- interpretation of results

3. Results

As stated in previous chapter, we investigate relationship between USA/Canada inflation differential and USD/CAD exchange rate. Initial proof of existence or nonexistence of this relationship is based on differential constructed on CPIs. Consequently, in order to verify achieved results, we construct differential by usage of GDP deflators and repeat the same testing procedure. Partial results both for procedure using CPIs and procedure using GDP deflators are stated simultaneously in order to save space.

The initial number of lags is set at fifteen, but the use of a sequential testing procedure shows that it is reasonable to include only one lag in case of all variables. The result is that variables with coefficients γ drop out from equation, therefore in order to test for existence of unit roots, we work with equation in following form:

$$\Delta Y_t = \alpha + \rho Y_{t-1} + \varepsilon_t \quad (15)$$

Because we have four variables (CPI differential, monthly exchange rate, deflator differential, quarterly exchange rate), we have four regressions and four different equations. Parameters of the executed regressions are stated below (constants turned out to be statistically insignificant and is dropped from equations).

Table 2: Dickey-Fuller Test Results for Each Variable

dependent var. (ΔY_t)	explanatory var. (Y_{t-}	coefficient (ρ	std. error	t-ratio	p-value	
d_dif_CPI	dif_CPI_1	-0,07419	0,02825	-2,63	0,0087	***
d_rate_m	rate_m_1	-1,07272	0,07333	-14,63	1.96e-028	***
d_dif_deflator	dif_deflator_1	-0,73215	0,10123	-7,23	4.09e-08	***
d_rate_q	rate_q_1	-0,84326	0,10500	-8,03	1.88e-020	***

Source: Author's own elaboration based on Gretl outputs

From t-ratios and p-values we can claim that all coefficients are statistically significant, in other words we decline hypotheses that they are equal to zero. And if $\rho \neq 0$, then $\phi \neq 1$, because $\rho = \phi - 1$. Therefore, each time series can be written in form

$$Y_t = \alpha + \phi Y_{t-1} + \varepsilon_t \quad (16)$$

It is proven that $\phi \neq 1$ and because condition $\phi = 1$ is a necessary feature of non-stationary time series, we can conclude that each time series under study has not unit root and therefore display stationary behavior.

After proof of stationarity of selected variables, we are free to perform VAR. First step is to find appropriate number of lags for each variable. The decision of most appropriate number of lags is based on information criteria – Akaike criterion (AIC), Schwarz Bayesian criterion (BIC) and Hannah-Quinn criterion (HQC). The best value of each criterion is always the lowest one. The highest number of lags was set on the level of 7, because it is highly unlikely that differences have long memory (it is the main issue of level variables), therefore inclusion of higher number of lags is not reasonable. All criteria suggest that the most appropriate is to include only 1 lag of variables in case of analysis based on CPI differential. The same result we get also in case of analysis based on deflator differential. Therefore the regression will take form of VAR(1).

Table 3: Lag Length Determination

		lags 1	lags 2	lags 3	lags 4	lags 5	lags 6	lags 7
CPI differential	AIC	**5,65985**	5,665249	5,707271	5,683188	5,70172	5,736397	5,721172
	BIC	**5,7308**	5,807158	5,920135	5,967006	6,056493	6,162125	6,217854
	HQC	**5,68862**	5,722787	5,793578	5,798264	5,845565	5,909012	5,922555
deflator differential	AIC	**8,52499**	8,573006	8,572197	8,627693	8,589961	8,599759	8,632266
	BIC	**8,69862**	8,862389	8,977333	9,148583	9,226603	9,352155	9,500415
	HQC	**8,59479**	8,689335	8,735058	8,837087	8,845886	8,902216	8,981255

Source: author's own elaboration based on Gretl outputs

Table 4 presents results from OLS estimation of VAR(1). Note that intercepts turn out to be statistically insignificant and for that reason they are not included in further analysis. Inclusion of time trend component improves results of regression only in third equation. Since there are two variables under study, there are two equations to estimate. Verification of findings based on CPI differential is done by another VAR(1) using deflator differential.

Each equations regresses a dependent variable on one lag of two variables in VAR. To save space, only selected characteristics are included in Table 4.

Table 4: The VAR(1) Using Changes of USD/CAD Exchange Rate and Inflation Differential as Dependent Variables

dependent variable	**explanatory variable**	**coefficient**	**t-ratio**	**p-value**			
rate_m	rate_m_1	-0,114878	-1,52	0,1304		R^2	0,083378
	dif_CPI_1	-0,481352	-2,174	0,0311	**	DW stat.	2,00488
dif_CPI	rate_m_1	0,0449613	4,455	1,50E-05	***	R^2	0,868266
	dif_CPI_1	0,938483	31,74	7,97E-74	***	DW stat.	2,024141
rate_q	rate_q_1	0,0892443	0,8281	0,41		R^2	0,116139
	dif_deflator_1	-0,631908	-2,107	0,0381	**	DW stat.	1,871503
	time	0,0280119	2,421	0,0176	**		
dif_deflator	rate_q_1	0,00753988	0,2184	0,8276		R^2	0,650266
	dif_deflator_1	0,474565	4,941	3.91e-06	***	DW stat.	2,203169

Source: author's own elaboration based on Gretl outputs

From above mentioned results of regression we can claim:

- Previous value of exchange rate change does not affect its value in next period. Coefficients are statistically insignificant which is proven by high p-values (0.1304 in case of regression using CPI differential, 0.41 in case of regression using deflator differential). Inclusions of these insignificant variables end up in low values of coefficients of determination.
- Value of inflation differential can be explained by its previous value. Coefficients are statistically significant which is proven by extremely low p-values (7.97E-74 in case of regression using CPI differential, 3.91e-06 in case of regression using deflator differential).
- Value of inflation differential can be explained by change in exchange rate in previous period, but only in case of regression using CPI differential (increase in exchange rate in previous period about one percentage point leads to increase in inflation differential in next period approximately about 0.0449 percentage point. However, this relationship is not found in case of regression using deflator differential. Therefore we cannot unanimously decide, whether this relationship exists or not.
- According to results, inflation differential explains change in exchange rate, but the effect is opposite than inflation differential theory suggests. According to regression, increase in differential in previous period about one percentage point leads to decrease of exchange rate in

next period approximately about 0.48 percentage point (in case of CPI differential), respectively about 0.63 percentage point (in case of deflator differential). From the view of USD, we are using direct quotation of exchange rate therefore if exchange rate is going down it means appreciation of US currency. According to theory, US currency should depreciate and therefore the coefficient should be positive.

The final conclusion is that inclusion of inflation differential alone is insufficient in order to explain change in USD/CAD exchange rate. The rate is most likely affected by other variables, and because these variables are not included in regression, our explanatory variable (inflation differential) take over their effect on exchange rate, which resulted in opposite sign of inflation differential coefficient. This shortcoming results in inability to prove or disprove the existence of inflation differential effect on USD/CAD exchange rate.

4. Conclusion

Possible existence of effect of inflation differential between USA and Canada on USD_CAD exchange rate was investigated in the presented paper. After brief theoretical insight into this issue, data and the methodology used in subsequent analysis were introduced. Several facts have been found in the empirical part of the paper. According to outputs of used VAR(1), it has been proven that previous value of exchange rate does not affect its value in following period. This is not the case of inflation differential, because it has been found out that inflation differential depends on its previous value. Effect of exchange rate on inflation differential is inconclusive.

However, our main goal was to find out, whether inflation differential affects exchange rate. We got into the contradiction with inflation differential theory, because according to our regression results, increase in inflation differential leads to appreciation of currency in subsequent period, instead of its depreciation. This result is explained by omission of variables, which can possibly affect exchange rate as well, therefore inflation differential took over effects of these omitted variables on exchange rate and its effect displays opposite trend. Because of this shortcoming, explanation of changes of exchange rate only by inflation differential does not seem as appropriate approach.

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Appendix

Time series plots of variables under study